Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced Subsidiary/Advanced Level

Monday 28 January 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP1), the paper reference (6667), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. Show, using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, that

$$\sum_{r=1}^{n} 3(2r-1)^2 = n(2n+1)(2n-1), \text{ for all positive integers } n.$$

(5)

2.
$$z = \frac{50}{3+4\text{i}}.$$

Find, in the form a + ib where $a, b \in \mathbb{R}$,

(a) z,

(2)

(b) z^2.

(2)

Find

(c) $|z|$,

(2)

(d) arg z^2, giving your answer in degrees to 1 decimal place.

(2)

3.
$$f(x) = 2x^{\frac{1}{2}} + x^{-\frac{1}{2}} - 5, \qquad x > 0.$$

(a) Find $f'(x)$.

(2)

The equation $f(x) = 0$ has a root α in the interval [4.5, 5.5].

(b) Using $x_0 = 5$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 significant figures.

(4)

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4. The transformation U, represented by the 2×2 matrix **P**, is a rotation through 90° anticlockwise about the origin.

(*a*) Write down the matrix **P**.

(1)

The transformation V, represented by the 2×2 matrix **Q**, is a reflection in the line $y = -x$.

(*b*) Write down the matrix **Q**.

(1)

Given that U followed by V is transformation T, which is represented by the matrix **R**,

(*c*) express **R** in terms of **P** and **Q**,

(1)

(*d*) find the matrix **R**,

(2)

(*e*) give a full geometrical description of T as a single transformation.

(2)

5.

$$\mathrm{f}(x) = (4x^2 + 9)(x^2 - 6x + 34).$$

(*a*) Find the four roots of $\mathrm{f}(x) = 0$.

Give your answers in the form $x = p + \mathrm{i}q$, where p and q are real.

(5)

(*b*) Show these four roots on a single Argand diagram.

(2)

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6. $$\mathbf{X} = \begin{pmatrix} 1 & a \\ 3 & 2 \end{pmatrix}, \text{ where } a \text{ is a constant.}$$

(*a*) Find the value of a for which the matrix $\mathbf{X}$ is singular.

(2)

$$\mathbf{Y} = \begin{pmatrix} 1 & -1 \\ 3 & 2 \end{pmatrix}.$$

(*b*) Find $\mathbf{Y}^{-1}$.

(2)

The transformation represented by $\mathbf{Y}$ maps the point A onto the point B.

Given that B has coordinates $(1 - \lambda, 7\lambda - 2)$, where λ is a constant,

(*c*) find, in terms of λ, the coordinates of point A.

(4)

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7. The rectangular hyperbola, H, has cartesian equation $xy = 25$.

The point $P\left(5p, \frac{5}{p}\right)$ and the point $Q\left(5q, \frac{5}{q}\right)$, where $p,\ q \neq 0$, $p \neq q$, are points on the rectangular hyperbola H.

(*a*) Show that the equation of the tangent at point P is

$$p^2y + x = 10p.$$

(4)

(*b*) Write down the equation of the tangent at point Q.

(1)

The tangents at P and Q meet at the point N.

Given $p + q \neq 0$,

(*c*) show that point N has coordinates $\left(\frac{10pq}{p+q}, \frac{10}{p+q}\right)$.

(4)

The line joining N to the origin is perpendicular to the line PQ.

(*d*) Find the value of p^2q^2.

(5)

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8. (*a*) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} r(r+3) = \frac{1}{3}n(n+1)(n+5).$$

(6)

(*b*) A sequence of positive integers is defined by

$$u_1 = 1,$$

$$u_{n+1} = u_n + n(3n+1), \quad n \in \mathbb{Z}^+.$$

Prove by induction that

$$u_n = n^2(n-1) + 1, \quad n \in \mathbb{Z}^+.$$

(5)

9.

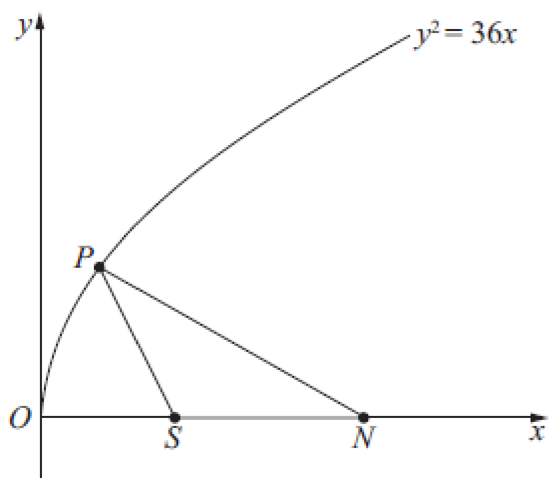


Figure 1

Figure 1 shows a sketch of part of the parabola with equation $y^2 = 36x$.

The point $P(4, 12)$ lies on the parabola.

(*a*) Find an equation for the normal to the parabola at P.

(5)

This normal meets the x-axis at the point N and S is the focus of the parabola, as shown in Figure 1.

(*b*) Find the area of triangle PSN.

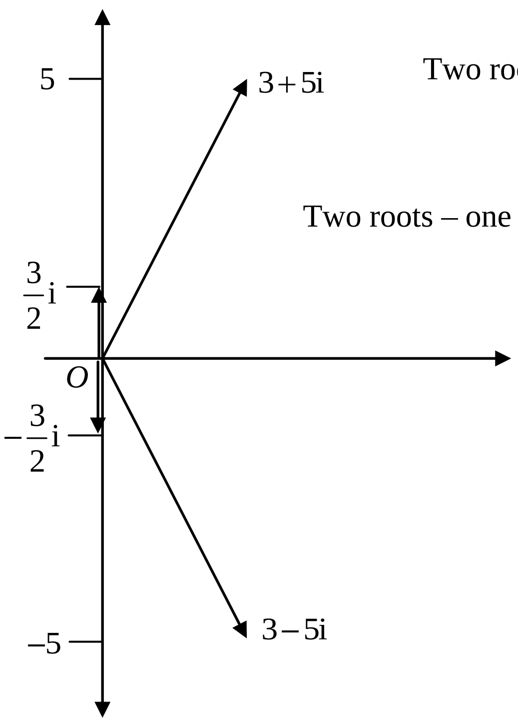
(4)

TOTAL FOR PAPER: 75 MARKS

END

EDEXCEL FURTHER PURE MATHEMATICS FP1 (6667) – JANUARY 2013 FINAL MARK SCHEME

Question Number	Scheme	Marks
1.	$\sum_{r=1}^{n} 3(4r^2-4r+1)=12\sum_{r=1}^{n} r^2-12\sum_{r=1}^{n} r+\sum_{r=1}^{n} 3$	M1
	$=\frac{12}{6}n(n+1)(2n+1)-\frac{12}{2}n(n+1), \quad +3n$	A1, B1
	$=n\left[2(n+1)(2n+1)-6(n+1)+3\right]$	M1
	$=n\left[4n^2-1\right]=n(2n+1)(2n-1)$	A1 cso
		[5]
2.	(a) $\frac{50}{3+4i}=\frac{50(3-4i)}{(3+4i)(3-4i)}=\frac{50(3-4i)}{25}=6-8i$	M1 A1cao **(2)**
	(b) $z^2=\left(6-8i\right)^2=36-64-96\text{i}=-28-96\text{i}$	M1 A1 **(2)**
	(c) $\lvert z\rvert=\sqrt{6^2+\left(-8\right)^2}=10$	M1 A1ft **(2)**
	(d) $\tan\alpha=\frac{-96}{-28}$	M1
	so $\alpha=-106.3^\circ$ or 253.7°	A1 cao **(2)**
		[8]
3.	(a) $\mathrm{f}'(x)=x^{-\frac{1}{2}}-\frac{1}{2}x^{-\frac{3}{2}}$	M1 A1**(2)**
	(b) f(5) = - 0.0807	B1
	$\mathrm{f}'(5)=0.4025$	M1
	$x_1=x_0-\frac{\mathrm{f}(x_0)}{\mathrm{f}'(x_0)}=5-\frac{-0.0807}{0.4025}$	M1
		A1
	=5.2(0)	**(4)**
		[6]

Question Number	Scheme	Marks
4.	(a) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	B1 **(1)**
	(b) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1 **(1)**
	(c) **R = QP**	B1 **(1)**
	(d) R $= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	M1 A1 cao **(2)**
	(e) Reflection in the *y* axis	B1 B1 **(2)** **[7]**
5.	(a) $4x^2+9=0 \quad \Rightarrow \quad x=k\text{i}, \quad x=\pm\frac{3}{2}\text{i}$ or equivalent	M1, A1
	Solving 3-term quadratic by formula or completion of the square $x=\dfrac{6\pm\sqrt{36-136}}{2}$ or $(x-3)^2-9+34=0$	M1
	$=3+5\text{i}$ and $3\text{-}5\text{i}$	A1 A1ft **(5)**
	(b) 	
	Two roots on imaginary axis	B1ft
	Two roots – one the conjugate of the other	B1ft
	Accept points or vectors	**(2)** **[7]**

Question Number	Scheme	Marks
6.	(a) Determinant: $2 - 3a = 0$ and solve for $a =$	M1
	So $a = \frac{2}{3}$ or equivalent	A1 **(2)**
	(b) Determinant: $(1\times 2)-(3\times -1)=5$ (Δ)	
	$Y^{-1}=\frac{1}{5}\begin{pmatrix}2 & 1\\ -3 & 1\end{pmatrix}$ $\left[=\begin{pmatrix}0.4 & 0.2\\ -0.6 & 0.2\end{pmatrix}\right]$	M1A1 **(2)**
	(c) $\frac{1}{5}\begin{pmatrix}2 & 1\\ -3 & 1\end{pmatrix}\begin{pmatrix}1-\lambda\\ 7\lambda-2\end{pmatrix}=\frac{1}{5}\begin{pmatrix}2-2\lambda+7\lambda-2\\ -3+3\lambda+7\lambda-2\end{pmatrix}=\begin{pmatrix}\lambda\\ 2\lambda-1\end{pmatrix}$	M1depM1A1 A1 **(4)** **[8]**

Question Number	Scheme	Marks
7.	(a) $y=\frac{25}{x}$ so $\frac{dy}{dx}=-25x^{-2}$	M1
	$\frac{dy}{dx}=-\frac{25}{(5p)^2}=-\frac{1}{p^2}$	A1
	$y-\frac{5}{p}=-\frac{1}{p^2}(x-5p) \Rightarrow \quad p^2y+x=10p \quad (*)$	M1 A1
		(4)
	(b) $q^2y+x=10q$ only	B1
		(1)
	(c) $(p^2-q^2)y=10(p-q)$ so $y=\frac{10(p-q)}{(p^2-q^2)}=\frac{10}{p+q}$	M1 A1cso
	$x=10p-p^2\frac{10}{p+q}=\frac{10pq}{p+q}$	M1 A1 cso
		(4)
	(d) Line PQ has gradient $\frac{\frac{5}{p}-\frac{5}{q}}{5p-5q}\left(=-\frac{1}{pq}\right)$	M1 A1
	ON has gradient $\frac{\frac{10}{p+q}}{\frac{10pq}{p+q}}\left(=\frac{1}{pq}\right)$ or $\frac{-1}{\frac{-1}{pq}}(=pq)$ could be as unsimplified	B1
	equivalents seen anywhere	
	As these lines are perpendicular $\frac{1}{pq}\times-\frac{1}{pq}=-1$ so $p^2q^2=1$	
	OR for ON	
	$y-y_1=m(x-x_1)$ with gradient (equivalent to) pq and sub in points O AND N to give $p^2q^2=1$	
	OR for PQ	
	$y-y_1=m(x-x_1)$ with gradient (equivalent to) $-pq$ and sub in points P AND Q to give $p^2q^2=1$	M1 A1
		(5)
		[14]

Question Number	Scheme	Marks
8.	(a) If $n=1$, $\sum_{r=1}^{n} r(r+3) = 1\times 4 = 4$ and	
	$\frac{1}{3}n(n+1)(n+5) = \frac{1}{3}\times 1\times 2\times 6 = 4$,	B1
	(so true for n = 1. Assume true for $n = k$)	
	So $\sum_{r=1}^{k+1} r(r+3) = \frac{1}{3}k(k+1)(k+5) + (k+1)(k+4)$	M1
	$= \frac{1}{3}(k+1)\left[k(k+5)+3(k+4)\right] =$	A1
	$\frac{1}{3}(k+1)\left[k^2+8k+12\right]$	
	$= \frac{1}{3}(k+1)(k+2)(k+6)$ which implies is true for $n=k+1$	dA1
	As result is true for $n=1$ this implies true for all positive integers and so result is true by induction	dM1A1cso
		(6)
	(b) $u_1 = 1^2(1-1)+1 = 1$	B1
	(so true for $n=1$. Assume true for $n = k$)	
	$u_{k+1} = k^2(k-1)+1+k(3k+1)$	M1,
	$= k(k^2-k+3k+1)+1 = k(k+1)^2+1$ which implies is true for $n = k+1$	A1
	As result is true for n = 1 this implies true for all positive integers and so result is true by induction	M1A1cso **(5)**
		[11]

Question Number	Scheme	Marks
9.	(a) $y = 6x^{\frac{1}{2}}$ so $\frac{dy}{dx} = 3x^{-\frac{1}{2}}$	M1
	Gradient when $x = 4$ is $\frac{3}{2}$ and gradient of normal is $-\frac{2}{3}$	M1 A1
	So equation of normal is $(y-12) = -\frac{2}{3}(x-4)$ (or $3y+2x=44$)	M1 A1
		(5)
	(b) S is at point (9,0)	B1
	N is at (22,0), found by substituting $y = 0$ into their part (a)	B1ft
	Both B marks can be implied or on diagram.	
	So area is $\frac{1}{2}\times 12\times(22-9) = 78$	M1 A1 cao
		(4)
		[9]